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Few-Body Systems

Few-Body Resonances in Light Nuclei

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Abstract. We have localized several few-body resonances in light nuclei, using methods which can properly handle two- or three-body resonant states. Among other results, we predict the existence of a three-neutron resonance, small spin-orbit splittings between the low-lying states in ^{5}He and ^{5}Li, the nonexistence of the soft dipole resonance in ^{6}He, new 1^+ states in ^{8}Li and ^{8}B, and the presence of a nonlinear amplification phenomenon in the 0_2^+ state of ^{12}C.

1 Introduction

In the past few years our theoretical understanding of the structure and reactions of light nuclei has been greatly improved, thanks to new and powerful methods and to the tremendous advances in computing power. Currently it is possible to solve the bound-state problems of $A \leq 8$ nuclei, using realistic two-body and three-body nucleon-nucleon (N-N) interactions, in a numerically exact way [1]. The scattering problem is more difficult to deal with. So far only the $A = 3$ systems can be treated with the same high precision as the bound states [2]. However, most of the states in light nuclei are unbound resonances. As the most elaborate models cannot treat these systems correctly for the time being, one can describe them either by using methods which are unphysical at some level, or by treating the most important degrees of freedom properly. We present here some of our recent results achieved by following the second strategy [3–12]. We concentrate mainly on the physics motivations and the most interesting results. Further details can be found in the original papers.

2 Model

We use a microscopic cluster model (RGM) description of nuclei, and apply this model to systems whose wave functions contain two- or three-cluster configurations with large weight. This ensures that those few degrees of freedom

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(the one or two relative motions between the clusters) which can be treated properly, are really the most important properties in the problems. The wave functions of the two- and three-cluster systems look like

$$\Psi = \sum_{L,S} \mathcal{A}\left\{ \left[\left[\Phi^A \Phi^B \right]_S \chi_L(\boldsymbol{\rho}) \right]_{JM} \right\} \tag{1}$$

and

$$\Psi = \sum_{l_1,l_2,L,S} \mathcal{A}\left\{ \left[\left[\Phi^A \Phi^B \Phi^C \right]_S \chi_{[l_1,l_2]L}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \right]_{JM} \right\}, \tag{2}$$

respectively. Here $\mathcal{A}$ is the intercluster antisymmetrizer, the Φ cluster internal states are translationally invariant $0s$ harmonic-oscillator shell-model states, the $\boldsymbol{\rho}$ vectors are the intercluster relative coordinates, l_1 and l_2 are the angular momenta of the two relative motions, L is the total orbital angular momentum, S is the total intrinsic spin, and [ ] denotes angular momentum coupling. In the case of three-cluster dynamics, all possible sets of relative coordinates [$A(BC)$, $C(AB)$, $B(AC)$] and angular momentum couplings are included in (2).

Putting (1) or (2) into the N-body Schrödinger equation, we get equations for the unknown relative motion functions χ. For two-body (three-body) bound states they are expanded in terms of (products of) Gaussian functions, and the expansion coefficients are determined from a variational principle for the energy. For two-body scattering states the χ functions are expanded in terms of Gaussian functions matched with the correct asymptotics, and the expansion coefficients are determined from the Kohn-Hulthén variational method for the S matrix [13].

In scattering theory resonances are defined as complex-energy solutions of the Schrödinger equation that correspond to the poles of the S matrix (or equivalently the zeros of the Fredholm determinant or Jost function). In order to obtain these complex solutions, we implemented a direct analytic continuation of the S matrix for two-cluster systems [7, 14], and the complex scaling method for three-cluster systems [15].

For two-cluster systems we solve the Schrödinger equation for the relative motion at complex energies with the boundary condition ($\rho \to \infty$)

$$\chi(\varepsilon, \rho) \to H^-(k\rho) - \tilde{S}(\varepsilon) H^+(k\rho). \tag{3}$$

Here ε and k are the *complex* energies and wave numbers of the relative motions, and H^- and H^+ are the incoming and outgoing Coulomb functions, respectively. The function $\tilde{S}$ has no physical meaning, except if it is singular at the energy ε. Then $\tilde{S}$ coincides with the physical S matrix, describing a purely outgoing solution, that is a resonance. So we search for the poles of $\tilde{S}$ at complex energies and extract the resonance parameters from $\varepsilon = E_{\rm r} - i\Gamma/2$.

For the three-cluster systems we solve the eigenvalue problem of a new Hamiltonian defined by

$$\widehat{H}_\theta = \widehat{U}(\theta) \widehat{H} \widehat{U}^{-1}(\theta), \tag{4}$$

where $\widehat{H}$ is the original many-body Hamiltonian and $\widehat{U}$ is the complex scaling transformation which acts on a function $f(\mathbf{r})$ as $\widehat{U}(\theta)f(\mathbf{r}) = e^{3i\theta/2}f(\mathbf{r}e^{i\theta})$. In the case of a multicluster system the transformation is performed on each dynamical coordinate (relative motion). The solution of the complex-scaled Schrödinger equation results in a spectrum with continuum cuts rotated by 2θ relative to the real energy axis, plus possibly a few isolated complex points at the resonant and bound state poles [15].

3 Resonances in $\mathbf{A = 2 - 12}$ Nuclei

We have used our model to study several selected resonances in $d(=p{+}n)$, ${}^3n(=n{+}n{+}n)$, ${}^3p(=p{+}p{+}p)$, ${}^3\mathrm{H}(=p{+}n{+}n)$, ${}^3\mathrm{He}(=p{+}p{+}n)$, ${}^4\mathrm{He}(=\{t{+}p, h{+}n\})$, ${}^5\mathrm{He}(=\alpha{+}n)$, ${}^5\mathrm{Li}(=\alpha{+}p)$, ${}^6\mathrm{He}(=\alpha{+}n{+}n)$, ${}^6\mathrm{Li}(=\alpha{+}p{+}n)$, ${}^6\mathrm{Be}(=\alpha{+}p{+}p)$, ${}^8\mathrm{Li}(=\alpha + t + n)$, ${}^8\mathrm{B}(=\alpha + h + p)$, and ${}^{12}\mathrm{C}(=\alpha + \alpha + \alpha)$ [3]. Here $\alpha = {}^4\mathrm{He}$, $t = {}^3\mathrm{H}$, and $h = {}^3\mathrm{He}$, and the cluster structures assumed in the model, are indicated. In most cases we used the Minnesota (MN) effective N-N interaction [16], which gives a reasonably good overall description of the low-energy $N+N$ scattering and the bulk properties of the ${}^3\mathrm{H}$, ${}^3\mathrm{He}$, and ${}^4\mathrm{He}$ clusters. In certain cases the Eikemeier-Hackenbroich (EH), modified Hasegawa-Nagata (MHN), or Volkov (V1 and V2) forces [17] were applied.

A=2: Our description of the virtual states in ${}^3\mathrm{H}$ and ${}^3\mathrm{He}$ (see below) requires the good reproduction of the virtual states of the $N + N$ systems in the ${}^1S_0$ channel. We localized these states on the complex-energy plane for the EH force by using the analytic continuation method [4]. In order to see how the differences between a neutral $(n + n)$ and a charged $(p + p)$ two-body virtual state develop, we first localized the $n + n$ pole and then smoothly switched on the Coulomb force. As one can see in Fig. 1, the presence of the Coulomb force creates two poles from the single virtual state present in $n{+}n$, and moves them into the complex plane. We note that in Ref. [4] our variational basis was not sufficiently converged, leading to slightly incorrect pole positions. We correct this error in Fig. 1. Our model gives the pole energies of the $n + n$ and $p + p$ states as $E_{nn} = -0.121$ MeV and $E_{pp} = (-0.143 \pm i0.466)$ MeV, in excellent agreement with the phenomenological values, $E_{nn} = -0.123$ MeV and $E_{pp} = (-0.140 \pm i0.467)$ MeV [18]. Our interaction is charge independent, leading to $E_{nn} = E_{np}$, therefore it is unable to reproduce the phenomenological $E_{np} = -0.066$ MeV value [18]. We would like to emphasize that, as Fig. 1 nicely demonstrates it, a virtual state with pure imaginary wave number can exist only in a neutral s-wave two-body system. The presence of a Coulomb-, centrifugal-, or three-body barrier does not allow the appearance of an S-matrix pole on the negative imaginary k axis.

A=3: The lightest nuclei where one could expect the existence of real resonances with any experimental significance are the $A = 3$ systems. The $3n$ and $3p$ nuclei are easier to handle, because the ${}^3S_1 - {}^3D_1$ two-nucleon channel is missing and there is no bound two-body subsystem present. We searched for three-body resonances in the various partial waves using the MN interaction,

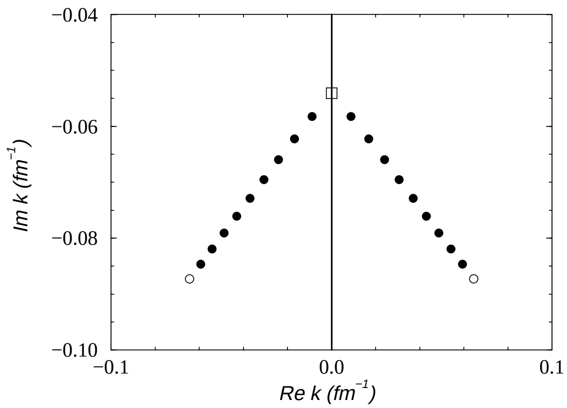

Figure 1. Trajectories of the $^1S_0$ $N{+}N$ S-matrix poles. The open square corresponds to the $n+n$ and $n+p$ poles, wile the open circles denote the pair of conjugate poles in the $p+p$ system. The filled circles come from calculations where $c \cdot V^{pp}_{\rm Coul}$ is added to the $n+n$ interaction $(0 < c < 1)$.

and found a $J^\pi = 3/2^+$ resonance in $3n$ with $E_r = 14$ MeV and $\Gamma = 13$ MeV parameters, while the mirror $3p$ system has $E_r = 15$ MeV and $\Gamma = 14$ MeV [5]. The EH interaction gives somewhat smaller resonance energies. We should mention that some recent experiments did not see any evidence of these structures [19]. Thus, it would be highly desirable to repeat our calculations using fully realistic forces. The first step in this direction has been made in Ref. [20]. So far those calculations could not be extended to the physical interactions, but the pole trajectories show that it is really the $3/2^+$ partial wave where one can expect a resonance, lying not very far from the real energy axis.

In ^{3}H and ^{3}He there are evidences for the existence of $1/2^+$ virtual states, both theoretically [21] and experimentally [22]. However, to our knowledge, all existing calculations so far were restricted to a simple configuration with a $^1S_0$ dinucleon plus the third nucleon. We extended those works by taking into account some other important channels, most importantly the $d{+}N$ configuration with a $^3S_1 - {^3D_1}$ deuteron. Our model, which works only below the three-body breakup threshold, gives $E_V = -1.62$ MeV and $E_V = (-0.43 \pm i0.56)$ MeV for the energies of the $1/2^+$ virtual states in ^{3}H and ^{3}He, respectively, using the EH force [5]. As the ground states of these nuclei are underbound in our model (due probably entirely to the lack of a three-body force), so are probably these virtual states. This underbinding leads to a too large $|E_V|$ for ^{3}H, and to ^{3}He states which lie too far from the imaginary k axis.

A=4: ^{4}He is the lightest nucleus with a well-established system of resonances. The first excited state, 0^+_2, which lies between the $^3\mathrm{H}+p$ and $^3\mathrm{He}+n$ thresholds, is perhaps one of the most difficult resonances to localize in ^{4}He. Those approaches which cannot use correct boundary conditions in their wave functions find this state several MeV above the 3+1 thresholds [23]. Our model reproduces the phenomenological $^1S_0$ phase shift in $^3\mathrm{H}+p$, which is the most important quantity related to the 0_2^+ resonance, rather well. We localize this state at $E_r = 93$ keV above the $^3\mathrm{H}+p$ threshold with $\Gamma = 390$ keV width, as a conventional resonance [6]. This result inspired us to repeat the search for this state in the so-called extended R-matrix-model description of the experimental data, because in the original search the resonance was not found [24]. This time we could find this state in the extended R-matrix model with $E_r = 114$ keV and $\Gamma = 392$ keV parameters, which are close to the RGM values.

A=5: The ^{5}He and ^{5}Li nuclei offer one of the cleanest and easiest testing grounds for resonance methods. Both of these systems have very strong $\alpha + N$ clustering nature, and there exist precise $\alpha+N$ scattering data. Still, there have been much controversy in the past few years concerning the resonances of ^{5}He and ^{5}Li. Shell models, for example, suggest the existence of low-lying positive parity states, most notably $1/2^+$ resonances [23]. Furthermore, it appears that the phenomenologically extracted spin-orbit splittings between the $3/2^-$ and $1/2^-$ states are so large that their theoretical reproduction is hopeless [25]. And, in general, there seems to be big differences between the resonance parameters coming from real-energy fits of certain reaction cross sections and those which are required by, e.g., the halo studies of ^{6}He.

Using our RGM model, we localized the low-lying ^{5}He and ^{5}Li states as poles of the scattering matrices [7]. Our calculations show that no low-lying $1/2^+$ state of any experimental significance exists in these nuclei. Extremely broad states ($\Gamma \gg 10$ MeV) can be found of course [26], as in almost any two-body system. However, they do not have any observable effect on the $\alpha + N$ scattering. The calculated $3/2^-$ and $1/2^-$ resonance parameters are somewhat different from those coming from conventional R-matrix analyses of the data [27], especially for the broader resonances. However, if the R matrices coming from the conventional data fit, are extended to complex energies, then we get a good agreement with our RGM results [7]. This finding emphasizes the fact that the correct treatment of the asymptotics in the analysis of experimental data (e.g. through the extended R-matrix method) can substantially affect the results of phenomenological analyses. Other calculations, using rather different models and interactions, and treating the resonances properly, found resonance parameters which are in excellent agreement with our results [28].

We should mention that the prediction of the $1/2^+$ state and other positive parity states by the shell model nicely demonstrates that the use of incorrect boundary conditions can lead to spurious states. In fact, if one could enlarge the basis size in such models beyond any limit, then all their predicted states (both the physical and spurious ones) would gradually move down to the lowest breakup threshold. One could distinguish between the physical states and the spurious ones by carefully analyzing their energy trajectories as functions of the basis size. The physical resonances would show up as those which remain stable for a relatively large interval of basis sizes [6, 26, 29].

A=6: Our main motivation of studying the resonances of the $A = 6$ nuclei was to see if the predicted soft dipole state exists in the neutron-halo nucleus ^{6}He. It was suggested that the oscillation of the halo neutrons against the ^{4}He core in ^{6}He would lead to a low-energy (a few MeV) dipole (1^-) resonance [30]. Break-up experiments performed on ^{6}He (and also on ^{11}Li) really indicate low-lying bumps in the dipole cross sections [31, 32], which means that a concentration of the dipole strength is undoubtedly present in these systems. Its origin is, however, questionable. Certain measurements show that these bumps come from a direct break-up process and not from a long-lived dipole state [32]. The main difficulty in interpreting the results comes from the fact that the experiments can see only a one-dimensional projection (on the real-energy axis) of a complex multisheeted energy surface, corresponding to the three-body problem. Trying to find out from this one-dimensional image if the scattering matrix has a 1^- (relative to the ground state) pole or not, is really difficult. Theoretically the situation is much easier, although those methods which are confined to real energies (e.g. the conventional shell model) face the same difficulty as the experimental analyses.

We searched for $\alpha + N + N$ resonances in ^{6}He, ^{6}Li, and ^{6}Be by using the complex scaling method in the cluster model [8]. We could find the experimentally known states, but we did not see any evidence for the existence of a 1^- state in ^{6}He. This result has been confirmed by other calculations using rather different methods and forces [33]. We should mention that those works, although do not see a 1^- state, indicate the existence of several previously unknown resonances in ^{6}He, like 0_2^+ and 1^+. We believe that these new states are real, although they do not show up in our model. It is possible that in our microscopic model these states would all be rather broad, which could explain why our method, which becomes unstable for broad states, cannot see them.

Recently, several rather narrow $\alpha + n + n$ resonances of ^{6}He were reported in each partial wave in Ref. [34]. This finding contradicts all previous works which, if they found any new state at all, indicated only a few rather broad new resonances. We believe that the Ref. [34] results are wrong and should be seen as a warning sign that the localization of resonances through the S-matrix poles, although a very powerful method, can generate false results if it is not done properly [9]. In Fig. 2 we show the complex-energy positions of the first four 1^- poles found in the Ref. [34] work (note that the first paper of Ref. [34] lists only the first two poles in each partial waves; the others can be found in the second one). The distribution of these poles is clearly unphysical.

A=8: The ^{7}Li$(n,\gamma)^8$Li and ^{7}Be$(p,\gamma)^8$B reactions play important roles in astrophysics. The first process takes place in certain inhomogeneous big-bang nucleosynthesis models [35], while the second one makes ^{8}B in the sun, which produces the highest-energy solar neutrinos with substantial flux [36]. Although astrophysically only the very low-energy (practically $E = 0$) cross sections are important, these values can be substantially influenced by the higher-energy continuum structures present in ^{8}Li and ^{8}B. For example, the extrapolation of the astrophysical S factor $S(E)$ of the ^{7}Be$(p,\gamma)^8$B reaction could be affected by the existence of a second 1^+ state at low energies [37]. With this motivation, we searched for low-energy 1^+ states in ^{8}Li and ^{8}B [10]. After tuning the MHN force to precisely reproduce the known parameters of the 0.632-MeV 1^+ state of ^{8}B (relative to ^{7}Be $+ p$), we indeed found new 1^+ states. In ^{8}B it is

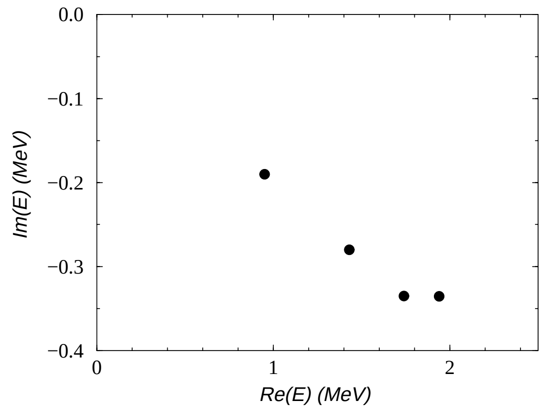

Figure 2. Positions of the the first four poles of the $J^\pi = 1^-$ $\alpha + n + n$ S matrix of Ref. [34] on the complex-energy plane.

situated at $E_r = 1.28$ MeV and has a $\Gamma = 0.56$ MeV width, while in ^{8}Li it lies right at the $^7\mathrm{Li} + n$ threshold. We would like to emphasize that so far there is no experimental evidence which would support our results. Nevertheless, given the important consequences of such states if they exist, we think that further experimental and theoretical studies of these possible structures would be desirable. One can see, e.g., that the presence of such a resonance would really affect the extrapolation of the experimental $S(E)$ of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction from higher energies down to $E = 0$ [10].

A=12: Recently, we studied the low-lying 3α resonances of ^{12}C [11]. We were able to reproduce the known resonances, and we believe that for the first time we showed that the 0_2^+ state is a genuine three-alpha resonance of ^{12}C. This level plays an important role in astrophysics, as virtually all the carbon in the Universe is synthesized through it [35]. But this 0_2^+ resonance is interesting for another reason, too. It possesses a rather curious feature, which we call nonlinear quantum amplifying [12]. If we change the strength of the N-N interaction by 0.1%, then the resonance energy of this state, relative to the 3α threshold, changes a lot more, by almost 10%. One can study this response to small perturbations also in other nuclei. It turns out that the relatively deeply bound states give a response which is comparable in size to the perturbation. However, as one moves close to the edge of stability, the effect of a small perturbation can get enormously amplified in the energy. This effect is caused by the fact that the residual interactions between the clusters, to which the nucleus breaks up, go toward zero much more mildly than the binding or resonance energy itself, as we go toward the break-up point.

This behavior is demonstrated in the case of the 0_2^+ state of ^{12}C in Fig. 3. We generated several artificial 0_2^+ states by changing the strength of the N-N force (multiplying all strength of the strong force by a number p). Then, for each artificial state (represented by its binding/resonance energy and its

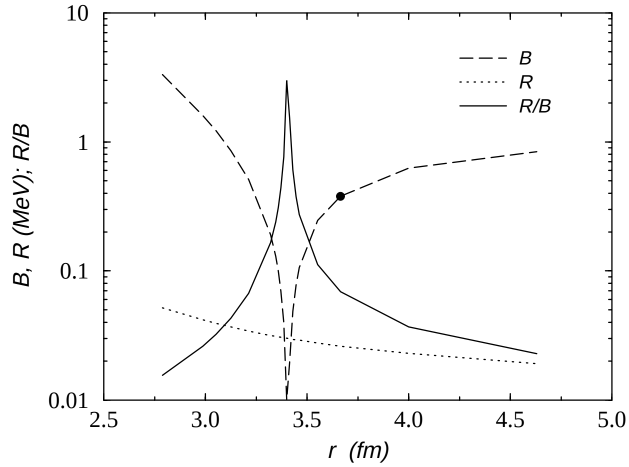

Figure 3. The energy ($B = |E|$, where E is the binding energy or resonance energy, relative to the breakup threshold), the response ($R = |E_p - E_{p\times 1.001}|$, where E_p and $E_{p\times 1.001}$ are the binding energies or resonance energies corresponding to a given N-N force and another one which is stronger by 0.1%, respectively), and the R/B ratio calculated for several artificial 0_2^+ states of ^{12}C, as functions of the radius of the state. The N-N interaction is chosen to be the MN force in each case, with the strengths multiplied by a number p (see the text). The black dot shows the real physical 0_2^+ state, given by our model.

radius) we calculated the response, that is the change of the binding/resonance energy caused by a 0.1% increase in the N-N strength. As one can see in Fig. 3, the response (which is closely related to the residual interaction) really behaves very differently than the binding/resonance energy, as we approach the breakup point. This naturally leads to the possibility of huge amplifications. We believe that this phenomenon is a common feature of nuclei lying at the edge of stability.

The strong sensitivity of the resonance energy of the 0_2^+ state of ^{12}C to the N-N force has a spectacular consequence in astrophysical carbon synthesis. Careful studies of all the details of the process show that a mere 0.5% change in the strength of the N-N force would lead to a Universe where virtually no carbon or oxygen exists [38]. This makes carbon production one of the most fine-tuned processes in astrophysics, leading to interesting consequences for the possible values of some fundamental parameters of the Standard Model [39].

4 Conclusions

We have presented selected examples of some interesting few-body resonances in light nuclei. We believe that the investigation of these and other resonance structures, using methods which can properly handle them, offers a rich source of information on many-body dynamics, nucleon-nucleon interaction, shell-structure, etc.

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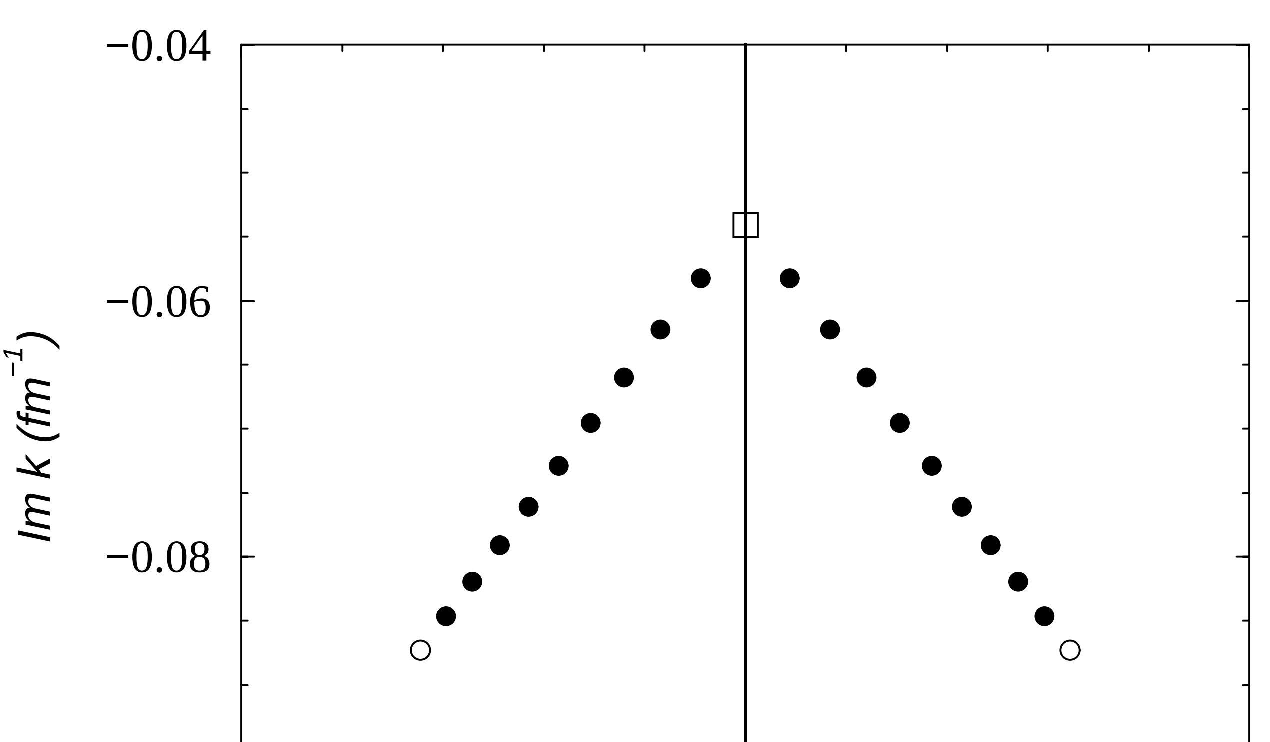

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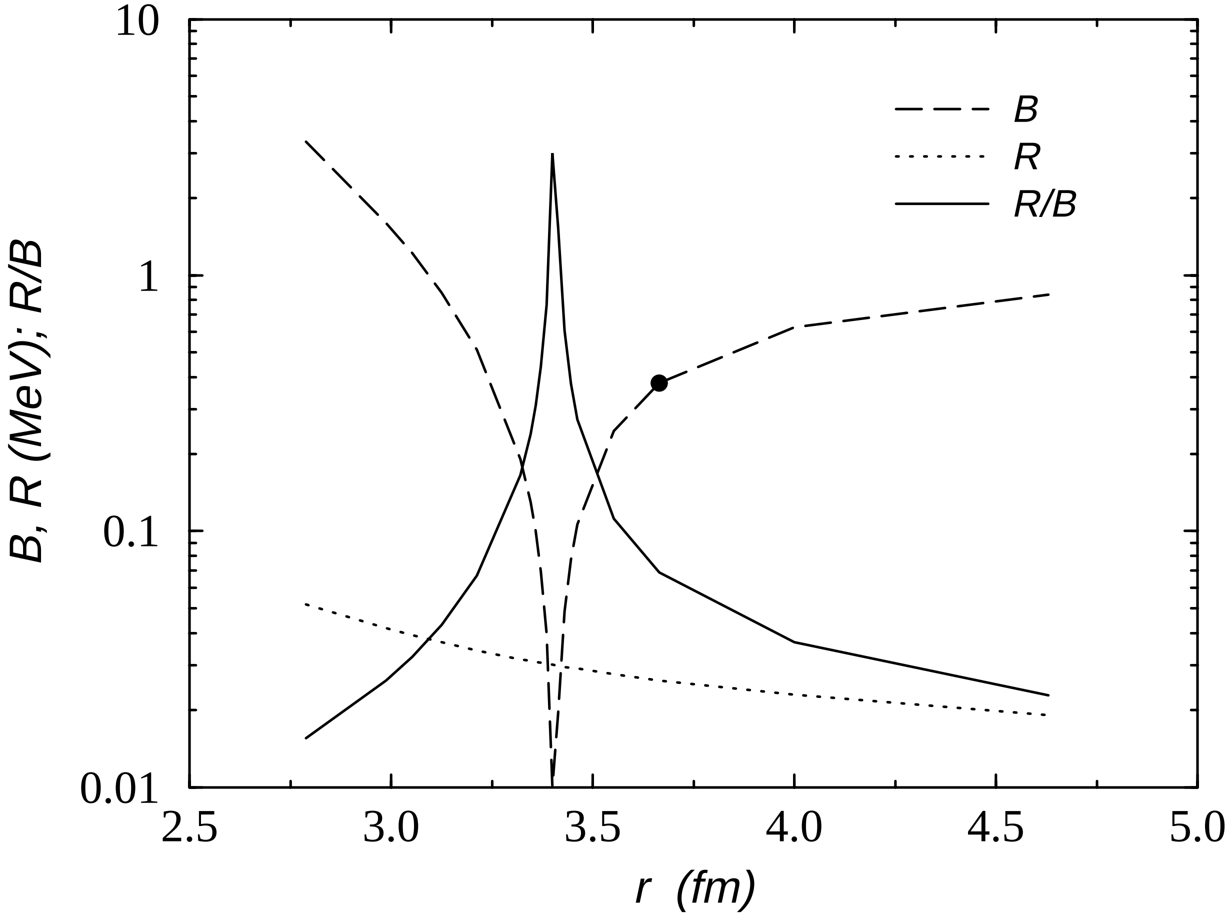

B
R
R/B
10
1
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r (fm)
B, R (MeV); R/B